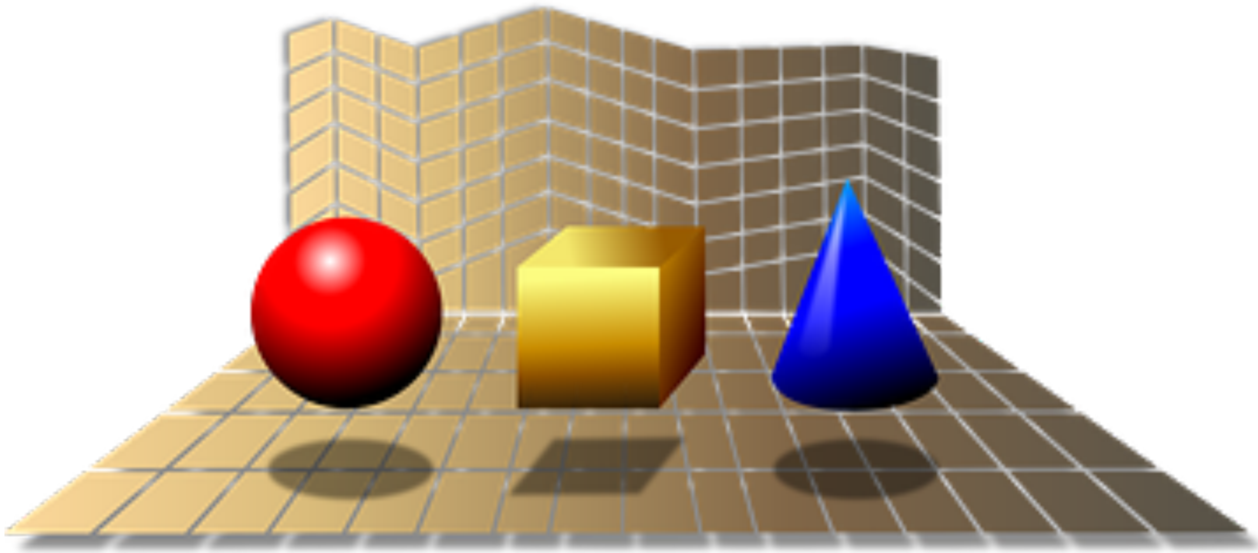
Advanced Functions & Modeling Indicators



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PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education | Department of Public Instruction

Advanced Functions and Modeling Objectives

Advanced Functions and Modeling provides students an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, and exponential functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

Strands: Data Analysis & Probability, Algebra

COMPETENCY GOAL 1: The learner will analyze data and apply probability concepts to solve problems.

Objectives

1.01 – Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems.

a) Interpret the constants, coefficients, and bases in the context of the data.
b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions and make predictions.

1.02 – Summarize and analyze univariate data to solve problems.

a) Apply and compare methods of data collection.
b) Apply statistical principles and methods in sample surveys.
c) Determine measures of central tendency and spread.
d) Recognize, define, and use the normal distribution curve.
e) Interpret graphical displays of univariate data.
f) Compare distributions of univariate data.

1.03 – Use theoretical and experimental probability to model and solve problems.

a) Use addition and multiplication principles.
b) Calculate and apply permutations and combinations.
c) Create and use simulations for probability models.
d) Find expected values and determine fairness.
e) Identify and use discrete random variables to solve problems.
f) Apply the Binomial Theorem.

COMPETENCY GOAL 2: The learner will use functions to solve problems.

Objectives

2.01 – Use logarithmic (common, natural) functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.02 – Use piecewise-defined functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.03 – Use power functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.04 – Use trigonometric (sine, cosine) functions to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
c) Develop and use the law of sines and the law of cosines.

2.05 – Use recursively-defined functions to model and solve problems.

a) Find the sum of a finite sequence.
b) Find the sum of an infinite sequence.
c) Determine whether a given series converges or diverges.
d) Translate between recursive and explicit representations.

AFM Objective 1.01
Calculator Models of Functions

Vocabulary/Concepts/Skills:

- Regression
- Residuals/Residual Plot
- Correlation Coefficient for linear data
- R^2
- Calculator Limitations with respect to data
- Interpret Constants, Coefficients and Bases
- Select the best model
- Interpolate
- Extrapolate
- Estimate
- Predict

Instructional Note: Knowing which type of regression to use is sometimes easily recognized and other times is difficult. Students should use residual plots for nonlinear regression to verify they have chosen the appropriate model. In general, a residual plot that seems random indicates a good fit. A residual plot that forms a pattern indicates another model may be more appropriate.
(Remember that the correlation coefficient is only valid for linear data.)

Technology Note: General directions for residual plots for most TI calculators:

- Turn the diagnostics on: Press **2nd** > **0** **[catalog]**; scroll down to **DiagnosticsOn** and hit **enter** (Remind students to do this prior to performing any regression so that they become familiar with the process prior to any major assessment.)
- Enter data into A list: Press **STAT** > **EDIT [1:Edit]**
- Perform the linear regression: Press **STAT** > **CALC [4: LinReg]**. The constants and coefficients of the equation should appear; along with the correlation (r) and coefficient of determination (r^2) values.
- To examine the residual plot to check the appropriateness of the model:
  - Store the residual values in **L3**: Return to the home screen → then press **2nd** > **STAT** **[list]** **[7: RESID];** then **STO→** > **2nd** > **3** **[L3]**
  - Examine the graph: Press **2nd** > **y=** **[stat plot]**; turn Plot1on and change the Ylist to L_3; then to graph press **Zoom** > **9: ZoomStat**.

Example 1: A ball is dropped over a motion detector and its height is recorded. The height is measured in feet and the time in seconds. The data is shown in the table.

Time (s)	**0**	**0.04**	**0.08**	**0.12**	**0.16**	**0.20**	**0.24**	**0.28**	**0.32**	**0.36**	**0.40**
Height (ft)	4.54	4.46	4.34	4.16	3.94	3.68	3.37	3.02	2.63	2.2	1.74

a. Create a scatter plot and determine the best model for the data.
b. Use the data to find a regression/prediction equation.
c. Predict the value if the time were .18 seconds? Is this an example of interpolation or extrapolation?
d. Compare the predicted value of .40 to the value given in the data.
e. State the difference in the preditcted value and the actual value, explain the meaning of the difference in context.
f. From your regression model, what are the x and y intercepts and what do they mean in context?

Example 2: A power function passes through the points $(1, 0.22)$ and $(6, 68)$.

a. Derive the power function that models this situation.
b. Based on your model, what is the value of the function when $x = 8$?
c. Based on your model, what is x when the value of the function is approximately 1569?

Example 3: In December 2003, a significant ice storm struck North Carolina resulting in numerous power outages. Because of the cold, wet weather and the extraordinarily large number of outages, it took many days for Duchess Power to restore electricity to its customers in Durham. The table below shows the number of days since the storm struck, the percent of customers whose electricity was restored that day, and the cumulative percent of customers whose power was restored.

Days without Power	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Percent Restored Daily	0.16	9.71	19.91	21.62	16.8	13.64	9.58	6.1	1.74	0.73	0.01
Total Percent Restored	0.16	9.87	29.78	51.4	68.2	81.84	91.42	97.52	99.26	99.99	100

a. Plot the data for days without power and total percent restored.
b. Identify the independent and dependent variables.
c. Estimate the graph of the function that fits this data and describe its end behavior. How does this make sense with the problem?
d. Find an appropriate function to model the data.
e. How does the data in Percent Restored Daily support your model choice?
f. Compare and discuss the actual values to the predicted values.

Example 4: Students were given a collection of number cubes. The instructions were to roll all of the number cubes, let them land on the floor, and then remove the number cubes showing FIVE. The students were told to repeat this process, each time removing all the Five's, until there were fewer than 50 number cubes left. The results are shown below.

Roll	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number Cubes Remaining	252	207	170	146	123	100	85	67	56	48

a. Create an exponential best-fit model for the data.
b. What characteristics of the data suggest that an exponential model is appropriate?
c. Based on your model, with how many number cubes did the students start?
d. Based on your model, describe the rate at which the number cubes are decreasing and the rate at which the number cubes is remaining.
e. Based on the model, how many times would the students have to roll the dice so that fewer than ten dice remained?

Example 5: The average price of a gallon of gas from 2008 to 2014 is given in the table.

Year	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
Average Price (dollars per gallon)	3.26	2.35	2.78	3.52	3.64	3.52	3.36

*U.S. Energy Information Administration / Monthly Energy Review January 2015

a. Create a scatter plot of the data.
b. Which type of function gives the best fit to the data?
c. Use the best-fit function to estimate the price per gallon of gas for 2007?
d. Is this an example of interpolation or extrapolation?

Example 6: The High Roller, in Las Vegas, Nevada, is the world's largest Ferris wheel. The wheel is approximately 550 feet in diameter. The wheel was designed to turn continuously and to be slow enough for people to hop on and off while it turns, completing a single rotation once every 30 minutes. Suppose you board this Ferris wheel at ground level. Let t represent the time since you boarded (in minutes) and let h represent your height above ground level (in feet).

a. Complete the table.

***t* (min)**	**0**	**7.5**	**15**	**22.5**	**30**	**37.5**	**45**	**52.5**	**60**
h (feet)	0								

b. Which function, sine or cosine, would best fit the data?
c. Find a regression model for the given data.
d. Describe the period and the amplitude.
e. Is there a phase shift or vertical shift applied to the model? If so, explain why it was necessary.

AFM Objective 1.02
Statistics

<u>Vocabulary/Concepts/Skills:</u>

- Measures of Central Tendency
- Measures of Variance
- Normal Distribution
- Standard Deviation
- Skewed right/Skewed left
- Random Sampling
- Census
- Survey
- Bias
- Population
- Various Graphical Representations
- Univariate Data
- Quantitative Data
- Simulation
- Experiment
- Observation
- Empirical Rule

Example 1: A company is wanting to know the effectiveness of a new treatment program for people who want to stop smoking.

a. How could the company use an observational study to determine effectiveness?
b. How could the company use an experimental study to determine effectiveness?
c. Which type of study, observational or experimental, would be better in this situation? Explain.

Example 2: During a presidental election, understanding statistics is essential.

a. Surveys are an important part of modern elections. What are some of the cautions one must take when setting up a survey?
b. How could the results of a survey be used to guide decisions about the time and resources of a campaign?

Example 3: Below are tables with results from the 2014 ACT.

States in which all graduates were tested	CO	IL	KY	LA	MI	MS	MT	NC	ND	TN	UT	WY		
Average composite score	20.6	20.7	19.9	19.2	20.1	19.0	20.5	18.9	20.6	19.8	20.8	20.1		
States in which less then 30% of graduates were tested	ME	RI	DE	PA	NH	MD	WA	MA	NJ	NY	VA	CA	CT	VT
Average composite score	23.6	22.9	23.2	22.7	24.2	22.6	23	24.3	23.1	23.4	22.8	22.3	24.2	23.2

Data Source: 2014 ACT National and State Scores http://www.act.org/newsroom/data/2014/states.html

a. Describe the distribution of each set of data.
b. Compare the appropriate measures of central tendancy and variation.
c. What conclusions, if any, can you draw from your findings?

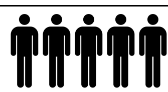
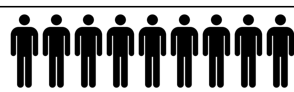
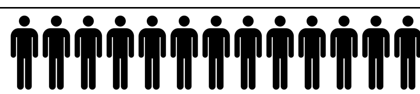
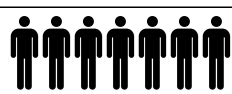
Example 4: A student needs to finish a class with at least an 80 in order to maintain a scholarship. The student has the following test scores in class: 75, 82, 79, 86, 89, 70, 74, and 76. The final grade is calculated by averaging the test scores and there is only one test left to take. (All test are out of 100 points.)

a. What is the lowest possible test score the student can earn to maintain the scholarship?
b. In order to make the deans list, the student must earn a 90 for this course. Is the dean's list a possibility for this student? Explain.

Example 5: The number of pages a print cartridge can print before needing to be replaced is normally distributed. The mean for a certain printer cartridge is 480 pages before needing to be replaced with a standard deviation of 20 pages. A large office building places a bulk order for 300 of those print cartridges.

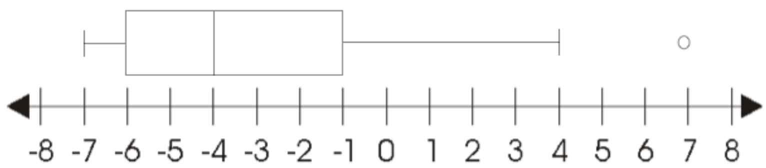
a. How many of the 300 print cartridges should be expected to print between 460 and 500 pages before needing to be replaced?
b. How many of the 300 print cartridges should be expected to print between 440 and 520 pages?

Example 6: The frequency chart below shows the number of males in a college course catagorized by height.

Height (inches)	Number males
51-55	
56-60	
61-65	
66-70	
71-75	
76-80	
81-85	

a. What is the shape of the distribution?
b. Estimate the mean and the median.
c. How might this chart and distribution be effected if the data for the females were included?

Example 7: Look at the box and whisker plot below and answer the following questions.



a. What information can you interpret from the graph?
b. How can you describe the distribution of the data graphed?
c. Considering the data set represented by the graph, describe at least two changes that would result in the median moving to -2.

AFM Objective 1.03
Probability

<u>Vocabulary/Concepts/Skills:</u>

- Counting
- Random
- Event
- Success/Failure
- Trial
- Sample Space
- Independent/Dependent
- Compound
- Mutually Exclusive
- Conditional
- Binomial Probability
- Expected Value
- Random Variable
- Fairness
- Simulation
- Combination
- Permutation
- Experimental Probability
- Theoretical Probability
- Discrete
- Continuous

Example 1: The student population at Roosevelt High is 1046. The entire student population was surveyed, and then categorized according to class and number of hours worked per week at a paying job.

	0 hr.	Work < 10 hr.	Work10 to 20	Work > 20 hr.
Freshmen	240	13	2	1
Sophomores	223	52	4	0
Juniors	103	25	88	47
Seniors	58	35	110	45

a. What is the probability that a randomly selected student from this school is a senior who does not have a job?
b. What is the probability that a randomly selected student is a sophomore who works between 10 and 20 hours per week?
c. What is the probability that a randomly selected student is a freshman?
d. What is the probability that a randomly selected student does not have a job?
e. What is the probability that a student is a freshman OR works less than 10 hours per week?
f. Which events are mutually exclusive?
   - Being a freshman and working less then 10 hours per week.
   - Being a senior and not having a job.
   - Being a sophomore and working more than 20 hours per week. What is the probability that a randomly selected student works more than 20 hours per week?

g. What is the probability that a randomly chosen student works more than 20 hours per week, given that s/he is a freshman?
h. What is the probability that a randomly chosen student works more than 20 hours per week, given that s/he is a senior?
i. Based on your last two answers, what comparison can you make between the freshman class and the senior class? Base your answer on the definition of independent events.
j. Determine whether or not a student's class (freshman, sophomore, junior or senior) and a student's work hours are independent of each other.

Example 2: A detective figures that he has a one in nine chance of recovering stolen property. His out-of-pocket expenses for the investigation are $6000. He is paid his fee only if he recovers the stolen property.

a. Write a statement that explains what should he charge clients in order to breakeven.

Example 3: A fair coin is tossed five times. On each toss, the probability of a head is $^1/_2$, and the five tosses are all independent events.

a . What is the probability that exactly two of the five coin tosses produced a head?
b . What is the probability that the five coin tosses produce at least one head?
c . At most one head?
d. What is the expected value of the number of heads?

Example 4: An unfair coin is weighted so that the probability of a head is $^1/_3$ and the probability of a tail is $^2/_3$. The coin is tossed seven times, and the outcome on each toss is independent of that on all of the other tosses.

a. What is the probability that the seven coin tosses produce at least two heads?
b . Exactly two heads?
c. Which is more likely, two heads out of seven or four heads out of seven? Justify your answer.

Example 5: Create a representation of the sample space that will show all of the possible outcomes of two randomly selected numbers between 0 and 8 in which repetition is allowed.

a. Create a probability distribution table for the sum of the two numbers.
b. What is the probability that their sum is less than or equal to five?
c. What is the probability that their sum is greater than or equal to nine?
d. What is the probability that their sum is 6 or 11?
e. What is the probability that their sum is 3 or 7?

Example 6: Each day two out of three teams are randomly selected to participate in a game.

a. What is the probability that team A is selected on at least two of the next three days?

Example 7: What is the fourth term for the expression of $(2x^3 + 4)^{10}$?

Example 8: A teacher is giving a 7 question true-false quiz. Some of the students were not prepared for the quiz and wanted to know what the probability was for a student to randomly guess at least 5 of the questions correctly to get a passing grade.

a. Design a simulation using appropriate technology and complete the chart below. Complete 30 trials.

   Use http://studenthelp.cpm.org/m/TI-84/l/95350-ti-84-generating-random-numbers to assist with running a simulation using a random number generator on a TI.

Number of correct answers	Tally Marks	Total
0		
1		
2		
3		
4		
5		
6		
7		

b. Based on your simulation, what is the probability for a student to randomly guess at least 5 of the 7 questions correctly?
c. Now compare your results with other students. How can you improve upon the results of the simulation?

Example 9: The table below shows the probability distribution of scores on the AP Calculus AB exam given during May of 2013.

s	1	2	3	4	5
$P(s)$	.294	.112	.173	.181	.239

Data Source: Student Score Distributions – AP Exams May 2013
http://media.collegeboard.com/digitalServices/pdf/research/2013/STUDENT-SCORE-DISTRIBUTIONS-2013.pdf

a. What is the probability that of a random student will score a 3 or higher?
b. At some universities, you must score a 4 or higher to be awards credit. What is the probability of a random student scoring 4 or higher?
c. 282,814 students took the AP Calculus AB Exam in May of 2013. How many students were not eligible to receive credit at a school that required a score of 3 or higher?
d. How many students could receive credit at school that required a 4 or higher?
e. What was the mean score for this exam?

Example 10: The student council conducted a poll to determine its activities for the year. 328 students responded to the poll.
Part of the survey asked about what dances the student council should organize: Homecoming Dance or a Winter Formal.

Dance	**Votes**
Homecoming	158
Winter Formal	127
Voted for Both	85

a. How many students did not vote for either dance?

Another part of the survey asked about the priorities of the student council. The students were given two options: Changing the dress code or getting more options for lunch in the cafeteria.

Priority	**Votes**
Changing Dress Code	257
More options for Lunch	198
Did not vote for a priority	15

b. How many students voted for both priorities?

Example 11: In a literature class, the students are going to be randomly assigned novel to read and write a report. The randomly assigned novel comes out of the teacher's library of 42 novels.

a. In a class of 27 students, how many combinations of novels and students are possible?
b. Before the books are assigned, the teacher tells the students that they will all get an A if anyone in the class can predict correctly which students are assigned which novel with the limit that each student only gets one guess. What is the probability of this occurring?
c. How could this problem have been rewritten so that order does not matter?

AFM Objective 2.01
Logarithmic Functions

<u>**Vocabulary/Concepts/Skills:**</u>

- Graph/Tables/Algebraic Properties
- Independent/Dependent
- Domain/Range
- Coefficients
- $y = a \cdot \log(bx + c) + d$
- $y = a \cdot \ln(bx + c) + d$
- Zeros
- Intercepts
- Asymptotes
- Increasing/decreasing
- Laws of Exponents
- Laws of Logarithms
- Global vs Local Behavior
- Continuous
- Discrete
- Solving Equations with justifications

Example 1: A study showed that the function $m(t) = 60\log(2t + 4)$ approximates the population of mice in a building abandoned 5 years ago with *t* being the number of months since the building was abandoned.

a. Use a table to find the population of the mice after each year. Which year showed the most growth? The least growth?
b. Describe the domain and range of the function.
c. How does using a logarithmic function fit the context of this problem?
d. When will $m(t) = 60$? How do you know?
e. Consider the graph of the function $m(t)$. What function would result by shifting the graph 3 units to the right? What would this mean in the context of this problem?

Example 2: An investment earning 8.5 % annually can be evaluated after *t* years, using $A(t) = A_n e^{0.085t}$.

a. When will a \$1200 investment be valued at \$1600?
b. At what rate would it double in 6 years?

Example 3: On September 25, 2006, Laurinburg NC experienced an earthquake that registered 3.7 on the Richter scale. The Richter scale was revised in 1979 so that *R*, the magnitude of the earthquake, is defined by $R = 0.67\log(0.37E) + 1.46$ where *E* is energy in kilowatt-hours (kWh)

a. How much energy was released in this earthquake?
b. According to the US Energy Information Administration, for 2013 the average US home used 10,908 kWh for that year. Based on that average, how many months does a US home use an equivalent amount of energy as this earthquake?
c. An Indian Ocean tsunami created on December 26, 2004 by an undersea earthquake was one of the largest earthquakes in recorded history. It measured 9.3 on the Richter scale. Based on the average energy used in a US home in a year, the energy from how many US homes is equivalent to the energy released during this earthquake? (There are approximately 123.2 million homes in 2014 in the US according to the US Census Bureau.)

Example 4: Solve for *x*:

a. $\log_2\left(\frac{x}{6}\right) = 5$
b. $\log_3 24 = x - 1$
c. $5\ln x - 3\ln x = 36$
d. $3e^{2x} - 5 = -2$

Example 5: Solve for *x* using technology.

a. $2\log_3(x-1) = \log_5(4x^2 - 25)$

AFM Objective 2.02
Piecewise-defined Functions

Vocabulary/Concepts/Skills:

- Graph
- Independent/Dependent
- Domain/Range
- Minimum/Maximum
- Increasing/Decreasing
- Global vs Local Behavior
- Continuous
- Discrete
- Solving Equations with justifications
- Interval notation

Example 1: Given the following piecewise function $h(x) = \begin{cases} x^2, & -3 \le x < 3 \\ 2 - x, & 3 \le x < 7 \end{cases}$

a. Sketch the graph and state the domain and range using interval notation.
b. What is $h\,(3)$? How do you know which function to use?

Example 2: A fast-moving cold front in the Northeast can cause temperatures to drop very quickly then rise again. The following data uses t as the hours since midnight on a day the cold front moves in, and T as the temperature in degrees Fahrenheit.

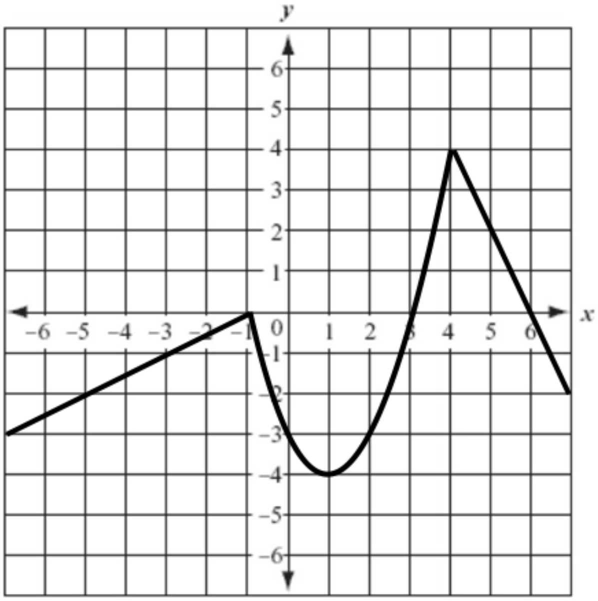
***t* (hr)**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
T (°F)	3	1	-1	-3	-5	-7	-5	-3	-1	1	3

a. Create a scatterplot of the data.
b. Write the functions and the corresponding domains that could be used to model this data.
c. Compare and contrast the interpretation of the coefficients and constants of both functions.
d. Another student decided to model this data with an absolute value function. Describe how you could create their model based from the graph of $f(x) = |x|$ and using transformations.

Example 3: Each orange tree in a California grove produces 600 oranges per year if no more than 20 trees are planted per acre. For each additional tree planted per acre, the yield per tree decreases by 7 oranges.

a. Describe the orange tree yield algebraically.
b. Determine how many trees per acre should be planted to obtain the greatest number of oranges.

Example 4: Write the piecewise function for the graph below.



Example 5: Using x to represent time and y to represent distance from home, create a piecewise function that represents a realistic morning exercise routine with the following criteria:
(Assume that the person is moving in a straight line away from and back to home.)

1) Starts out walking from home for 5 minutes
2) Jogs for the next 20 minutes
3) Stops to get a drink of water for 1 minute
4) Turns around and jogs back towards home for 15 minutes
5) Walks back home for 10 minutes.

AFM Objective 2.03
Power Functions

Vocabulary/Concepts/Skills:

- Graph
- Independent/Dependent
- Domain/Range
- Coefficients
- $y = a \cdot x^b + c$
- Zeros
- Intercepts
- Asymptotes
- Minimum/Maximum
- Increasing/Decreasing
- Continuous
- Discrete
- Solving Equations with justifications
- End Behavior

Example 1: The table below represents the amount of bacteria over a given period of time.

Time in minutes (x)	**1**	**4**	**7**	**9**	**13**	**17**	**20**	**23**	**25**
Number of bacteria (y)	3	21	46	65	108	158	198	240	270

a. Write a power regression equation ($y = a \cdot x^b$) for the growth of the bacteria. (Round values to the nearest thousandths.)
b. In your regression equation, describe what does *a* represent and how does *b* indicate the end behavior?
c. Predict the amount of bacteria at 15 min? 28 min?
d. Which of the predictions from c is an example of extrapolation and which is an example of interpolation?

Example 2: The community recreation center is draining the pool for the winter. Several students collected data during the time the water was draining. The table below shows the amount of water during left in the pool 30-minute intervals.

30 minute intervals	**1**	**2**	**3**	**4**	**9**	**10**
Amount of water (gallons)	400.2	224.28	160.06	125.78	64.05	58.58

a. Write a power regression equation ($y = a \cdot x^b$) for the water left in the pool draining from the pool. (Round to the nearest thousandth.)
b. In your regression equation, describe what does *a* represent and how does *b* indicate the end behavior?
c. How much water is remaining after 3 hours? 6 hours?
d. How many gallons of water were initially in the pool? What are the challenges with your regression equation and this question?

Example 3: Solve the following equation: $3x^{2.2} = 6$

Example 4: Given $y = 4x^{\frac{1}{2}}$

a. Determine if there are any asymptotes and if so state them.
b. Create a problem that this function could model.

AFM Objective 2.04
Trigonometric Functions

<u>Vocabulary/Concepts/Skills:</u>

- Graph
- Independent/Dependent
- Domain/Range
- Period
- Amplitude
- Phase shift
- Vertical Shift
- Frequency
- Coefficients
- $y = a \cdot \sin(bx + c) + d$
- $y = a \cdot \cos(bx + c) + d$
- Intercepts
- Law of Sines
- Law of Cosines
- Unit Circle
- Radian/Degree Measure
- Special angles (multiples of $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$)
- Solving Equations with justifications

Example 1: Sketch the graph of $f(x) = a \cdot \cos(bx) + c$ for $0 < x < 2$ where $a = 5$, $b = 2$, and $c = 3$.

a. Discuss the intercepts and the maximum and minimum points.
b. As the value of *a* approaches zero, explain the change in value of $f\ (x)$.
c. As the value of *b* increases, explain the change to the corresponding graph.

Example 2: Consider the functions $f(x) = 3 \cdot \sin\left(\frac{1}{2}x\right)$ and it's parent function $g(x) = \sin x$.

a. Which function has the larger amplitude?
b. Write a function that has a smaller amplitude then either $f\ (x)$ or $g\ (x)$.
c. Write a function that has a longer period then either $f\ (x)$ or $g\ (x)$.
d. If $k(x) = 3 \cdot \sin\left(\frac{1}{2}x - \frac{\pi}{2}\right) + 1$, describe the transformation from $f\ (x)$.

Example 3: A supporting post on a pier shows the height of the water as 5.4 feet at low tide (2AM) and a height of 11.8 feet at the next high tide, 6.2 hours later.

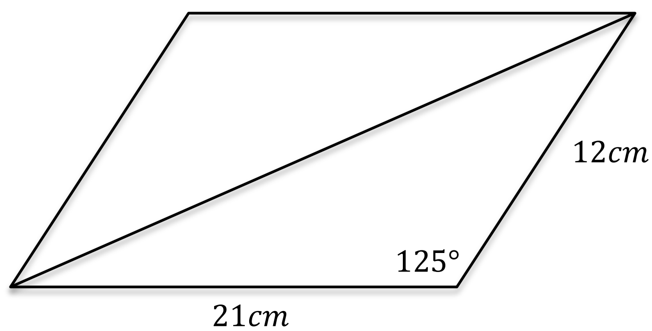
a. Write an equation describing the depth of the water at this location *t* hours after midnight.
b. What will be the depth of the water at this support at 4 AM?

Example 4: The height of a seat on a Ferris wheel with a diameter of 14 meters, *t* seconds after it begins to turn at 3 rpm, can be computed using this sinusoidal model, $H(t) = 7cos\left[\frac{2\pi}{20}(t - 10)\right] + 8$.

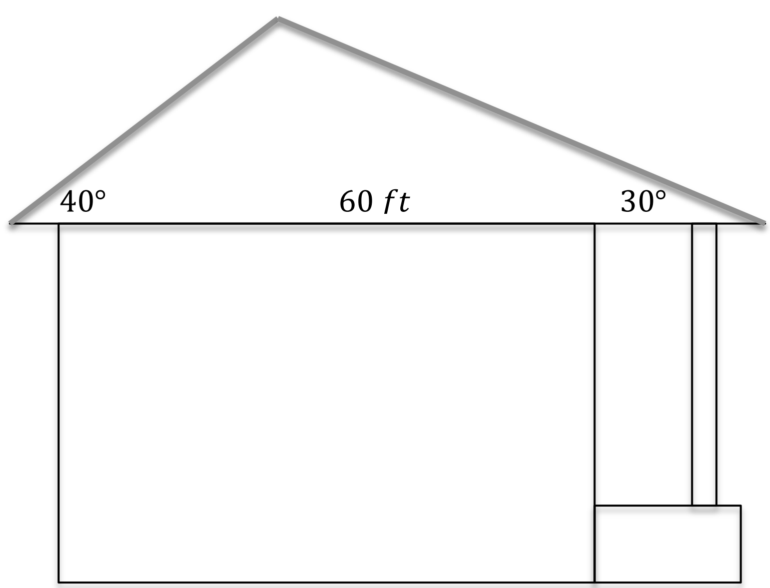
a. Graph the function.
b. When will the height of the seat be a minimum? A maximum? What are these heights?
c. If the wheel is replaced with one with a larger diameter, 16 feet, how would the parameters in the equation be affected? Test your conjecture on the calculator.
d. If the original wheel turns at 4 rpm, which parameter(s) are affected? Graph your altered equation to check the outcome.

Example 5: A jet is flying over a long, thin island just off the coast at an altitude of 30,000 ft. The copilot notes that when looking directly out the right window, one tip of the island has an angle of depression of about 50° and that when looking directly out the left window the other tip of the island has an angle of depression of about 35°. About how many miles is the distance from one tip of the island to the other?

Example 6: What is the length of the diagonal of the parallelogram below?



Example 7: A roofer needs to estimate the area of a roof of a home. Use the information from the side view of the home below to find the area of the roof. The home is 180 ft long.



AFM Objective 2.05
Recursively-defined Functions

Vocabulary/Concepts/Skills:

- Arithmetic Sequence
- Geometric Sequence
- Geometric Series
- Subscript Notation
- Summation Notation
- Converge/Diverge
- Limit
- Translate between Recursive and Explicit Representations

Example 1: The Tower of Hanoi puzzles consist of a stack of wooden disks of graduated sizes on one of three wooden pins. One may move only one disk at a time, and never put a larger disk onto a smaller disk. The goal is to move all the disks to another pin in a minimum number of moves.

a. Use the link http://haubergs.com/hanoi to calculate the minimum number of moves that it takes to relocate *n* disks and complete the table.

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
S(n)								

b. Find a recursive function of the form $S(n+1) = a \cdot S(n) + b$ to fit this data.
c. Find an explicit formula for this data. It is of the form $S(n) = b^n + c$.
d. Confirm the results in your table.
e. How many steps would it take to move 20 disks?

Example 2: A patient takes 800 mg of ibuprofen for pain. During any 4hr time period his body will metabolize 35% of the medicine.

a. If the patient takes a dose of medication at 8am and did not repeat the dose, how much medication will be in his blood stream at midnight?
b. Suppose that same patient repeats that same 800 mg dose every 4hrs, how much medicine will be in his blood stream before he takes his dose at midnight?

Example 3: A sequence is shown below. $1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \ldots$

a. What is the sum of the sequence?

Example 4: Create an example of a two series in which one converges and the other diverges. Explain how you know your example fit the criteria for convergence and divergence.

Example 5: In an arithmetic sequence, $a_1 = 3$ and the common difference is 7. What is the sum of the first 25 terms?

Example 6: A student is starting an exercise routine with the goal of running a marathon in 60 days. The student wants to track the time spent in preparation for the marathon. The student starts by running for ½ hour and then running an additional 5 minutes per day. If the student is able to maintain this exercise routine, how many hours of preparation will the student have accumulated 5 days before the race?